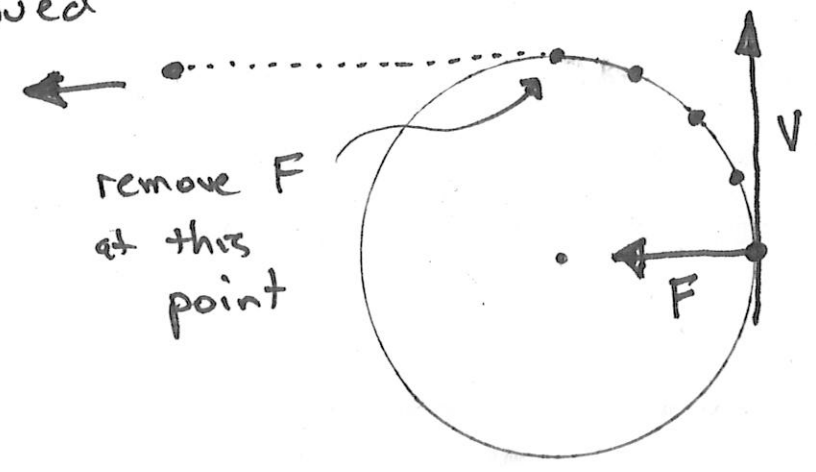
Circular Motion

Newton's first law → an object in motion stays in motion unless acted upon by an outside force

This means motion in a straight line!!

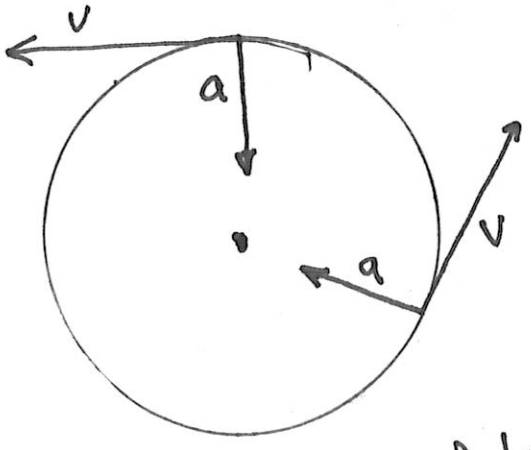
In circular motion the direction is continuously changing ∴ a force must continuously be pulling the object toward a center. The velocity of the object, at any point in time is considered to be tangent to the circular path. If the force is removed the object will continue in a straight path from the point at which the force was removed



Remember that acceleration results from a change in velocity. But velocity is a vector with magnitude and direction. In circular motion the magnitude may remain constant but the direction does not. We therefore have a constant acceleration to maintain a circular path. Since $F = ma$ and the mass is constant, the direction must constantly be changing toward the center.

This acceleration is called <u>Centripetal acceleration</u> (center-seeking) and is directed along the radius toward the center of the circle

$$a_c = \frac{v^2}{r}$$



The acceleration vector points towards the center but the velocity vector always points in the direction of motion which is tangential to the circle

Note $a_c \propto v^2$ so double v 4x acceleration

VII. CIRCULAR MOTION

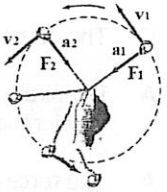
POINT 1 Uniform Circular Motion

A. Uniform circular motion is the motion of an object moving at constant speed in a circle with a fixed radius.

B. The period, T, of the motion is the time required to make one revolution, and the frequency, f, is the number of revolutions per second.

$$T = \frac{1}{f} \qquad \text{speed: } v = \frac{2\pi r}{T}$$

C. In uniform circular motion, the velocity and the acceleration are continually changing direction, and are perpendicular to each other at each moment. The force on the stone swung on the end of a string is the force exerted inwardly by the string. When it is released, the stone flies off tangentially, in the direction of the velocity it has at the moment.



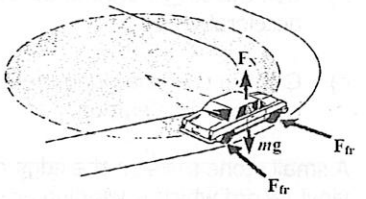
D. In uniform circular motion, the centripetal acceleration, a_C, of an object is directed toward the center of the circle and continually changes direction as the object moves.

$$a_C = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

E. The centripetal force, F_C, toward the center of the circle is the net force required to keep an object revolving in a circle.

$$F_C = ma_C = m\frac{v^2}{r} = m\frac{4\pi^2 r}{T^2}$$

F. Centripetal force can be provided by the tension in a cord, the friction between a road and tires, or the normal force by a wall. For example, when a car rounds a flat curve, the friction between the road and the tires provides the centripetal force to keep the car on the road.

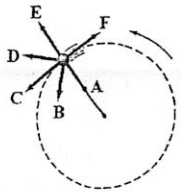


$$F_N = mg \qquad F_{fr} = \mu \cdot F_N = \mu \cdot mg$$

$$F_C = F_{fr} \qquad \frac{mv^2}{r} = \mu \cdot mg \qquad v = \sqrt{\mu g r}$$

PROBLEM 1 Uniform Circular Motion

A ball is swung on the end of a string at constant speed in a horizontal circle.



a) Which path does the ball follow at the moment the string breaks?
b) Which path represents the direction of the velocity of the ball?
c) Which path represents the direction of the acceleration of the ball?
d) Which path represents the direction of the force exerted on the ball by the string?

Solution

a) C When the string breaks, the ball flies off tangentially in the direction of the velocity it has at the moment.

b) C The direction of the instantaneous velocity of the ball is in a direction tangent to the circular path.

c) A In uniform circular motion, the acceleration of the ball is directed toward the center of the circle. This acceleration is a centripetal acceleration.

d) A The force exerted on the ball by the string is directed toward the center of the circle. This force is a centripetal force.

RELATED PROBLEMS

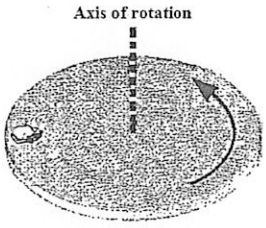
1. A car is about to go around a curve. Using principles of physics, explain the following questions.

   a) Can the car go around the curve with constant speed?

   b) Can the car go around the curve with zero acceleration?

   c) Can the car go around the curve with constant acceleration?

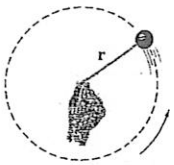
2. A small stone rests on the edge of an antique vinyl record which is whirling around at about 33 rotations per minute.



   a) Draw a diagram showing the forces acting on the stone.

   b) What provides the centripetal force to keep the stone on the record?

3. A 0.20 kg ball at the end of a string with a radius of r is revolving uniformly in a horizontal circle.



   a) What provides the centripetal force to keep the ball in circular motion?

   b) Sketch the graph showing the relationship between the centripetal acceleration, a_C, and the velocity, v. There is no need to plot any data point.

   c) Sketch the graph showing the relationship between the centripetal force, F_C, and the radius of the circle, r. There is no need to plot any data point.

   d) If the radius of the circle is doubled and the speed remains the same, how does this change affect the centripetal force?

   e) If the speed of the ball is doubled and the radius of the circle remains the same, how does this change affect the centripetal force?