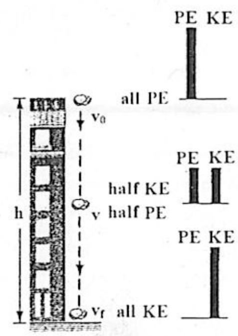
POINT 3 Conservation of Mechanical Energy

A. The total mechanical energy of an object remains constant in the absence of nonconservative forces such as friction. While the total mechanical energy is constant, the kinetic energy and the gravitational potential energy may be transformed into each other.

$E_0 = E_f$ $\quad KE_0 + PE_0 = KE_f + PE_f$ $\quad \frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_f^2 + mgh_f$

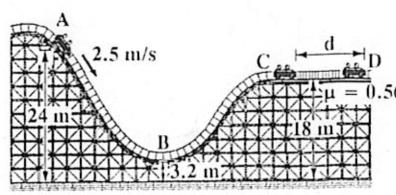


B. If nonconservative forces such as friction act on an object, the mechanical energy is not conserved.

$$KE_0 + PE_0 = KE_f + PE_f + F_{fr}d$$
$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_f^2 + mgh_f + F_{fr}d$$

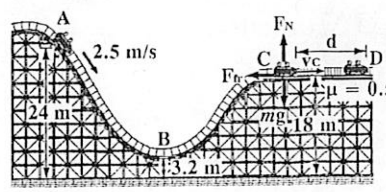
PROBLEM 3 Conservation of Mechanical Energy

A roller coaster car moving at 2.5 m/s slides from A to C along a frictionless track, and then passes through a horizontal track from C to D, where a frictional force acts on it as shown in the diagram. As a result, the car slows down, and comes to a stop at point D. The coefficient of kinetic friction between the car and the track in the region CD is 0.56. Ignore air resistance.



a) Find the speed of the car at points B and C.
b) How far does the car move from point C before coming to a stop at point D?

Solution $\quad v_A = 2.5$ m/s, $h_A = 24$ m, $h_B = 3.2$ m, $h_C = h_D = 18$ m, $\mu = 0.56$



a) $\mathbf{KE_A + PE_A = KE_B + PE_B}$ $\quad \frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$

$$\frac{1}{2}v_A^2 + gh_A = \frac{1}{2}v_B^2 + gh_B$$

$$\frac{1}{2}(2.5)^2 + (9.8)(24) = \frac{1}{2}v_B^2 + (9.8)(3.2) \qquad v_B = 20 \text{ m/s}$$

$\mathbf{KE_A + PE_A = KE_C + PE_C}$ $\quad \frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_C^2 + mgh_C$ $\quad \frac{1}{2}v_A^2 + gh_A = \frac{1}{2}v_C^2 + gh_C$

$$\frac{1}{2}(2.5)^2 + (9.8)(24) = \frac{1}{2}v_C^2 + (9.8)(18) \qquad v_C = 11 \text{ m/s}$$

b) $v_C = 11$ m/s, $v_D = 0$ m/s

$F_N - mg = 0$ $\quad F_N = mg$ $\quad F_{fr} = \mu F_N = \mu mg$

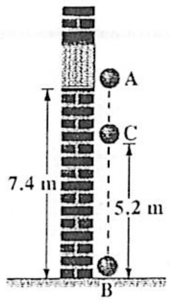
$\mathbf{W_{fr} = -F_{fr}d} = -(\mu mg)d$ $\quad \mathbf{W_{fr} = \Delta KE_{CD}} = KE_D - KE_C = \frac{1}{2}mv_D^2 - \frac{1}{2}mv_C^2$

$-(\mu mg)d = \frac{1}{2}mv_D^2 - \frac{1}{2}mv_C^2$ $\quad -\mu gd = \frac{1}{2}v_D^2 - \frac{1}{2}v_C^2$

$$-(0.56)(9.8)d = \frac{1}{2}(0)^2 - \frac{1}{2}(11)^2 \qquad d = 11 \text{ m}$$

RELATED PROBLEMS

13. A basketball is dropped out of a window 7.4 m above the ground. The basketball hits the ground and rebounds to 5.2 m. Ignore air resistance.

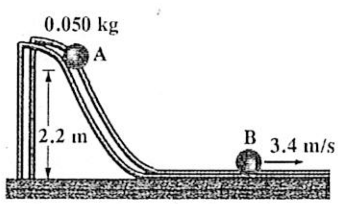


a) Find the speed of the basketball just before it hits the ground.

b) Find the speed of the basketball when it rebounds.

c) If the mass of the basketball is 0.62 kg, find the change in mechanical energy of the basketball when it hits the ground and rebounds.

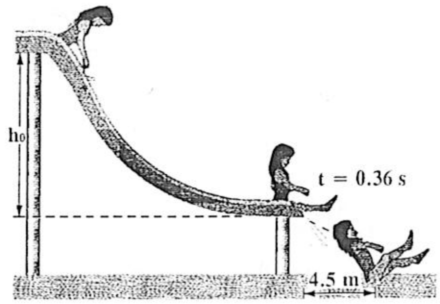
14. A 0.050-kg ball starts from rest at point A and slides down a track 2.2 m above the ground as shown in the diagram. The ball reaches a speed of 3.4 m/s at point B. The length of the track from point A to point B is 4.6 m.



a) Find and explain the change in the mechanical energy of the ball from point A to point B.

b) Find the average force of friction acting on the ball from point A to point B.

15. A student starts from rest at the top of a water slide. After leaving the horizontal bottom of the slide, she lands in the water 4.5 m from the edge of the slide in 0.36 s. Ignore friction and air resistance.



a) Find the height of the slide from which she starts.

b) The student slides down another water slide with the same height as the previous one, but with a uniform slope. Explain how the speed at the bottom of the new slide compares with that of the previous one.