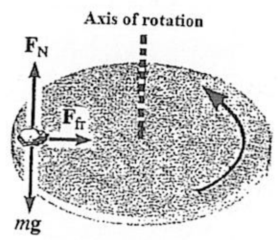
VII. CIRCULAR MOTION

1. a) Since the magnitude of velocity (speed) may be constant, the car can go around the curve with constant speed.

   b) Since the direction of the velocity of the car changes as the car goes around a curve, the acceleration of the car cannot be zero.

   c) Since the direction of the acceleration of the car changes, the car cannot go around the curve with constant acceleration.

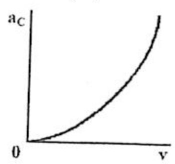
2. a) Diagram showing the forces acting on the stone



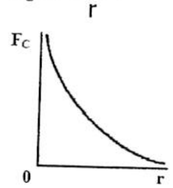
   b) The static friction between the record and the stone provides the centripetal force.

3. a) The tension of the string provides the centripetal force to keep the ball in circular motion.

   b) $a_C = \left(\frac{1}{r}\right)v^2$



   c) $F_C = \dfrac{mv^2}{r}$



   d) $r_2 = 2r_1$

$$F_{C1} = \frac{mv^2}{r_1}$$

$$F_{C2} = \frac{mv^2}{r_2} = \frac{mv^2}{2r_1} = \frac{1}{2}\left(\frac{mv^2}{r_1}\right) = \frac{1}{2}F_{C1}$$

   The centripetal force is half as great as before.

   e) $v_2 = 2v_1$

$$F_{C1} = \frac{mv_1^{\,2}}{r}$$

$$F_{C2} = \frac{mv_2^{\,2}}{r} = \frac{m(2v_1)^2}{r} = 4\left(\frac{mv_1^{\,2}}{r}\right) = 4F_{C1}$$

   The centripetal force is 4 times as great as before.

4. $m = 0.20$ kg, $r = 0.60$ m, $T = 1.2$ s

a) $a_C = \dfrac{4\pi^2 r}{T^2} = \dfrac{4\pi^2(0.60)}{(1.2)^2} = 16.45 \approx 16\ \text{m/s}^2$

b) The tension in the string provides the centripetal force on the stone.

$F_C = F_T \qquad F_T = ma_C = (0.20)(16.45) = 3.3\ \text{N}$

5. $r = 60$ m

a) $v_D = 18$ m/s

$F_N = mg \qquad F_{fr} = \mu_D \cdot F_N = \mu_D \cdot mg$

$F_C = F_{fr} \qquad \dfrac{mv_D^2}{r} = \mu_D \cdot mg \qquad \dfrac{v_D^2}{r} = \mu_D g \qquad \dfrac{(18)^2}{60} = (9.8)\mu_D \qquad \mu_D = 0.55$

b) $\mu_W = \mu_D(0.4) = (0.55)(0.4) = 0.22$

$F_N = mg \qquad F_{fr} = \mu_W \cdot F_N = \mu_W \cdot mg$

$F_C = F_{fr} \qquad \dfrac{mv_W^2}{r} = \mu_W \cdot mg \qquad \dfrac{v_W^2}{r} = \mu_W g \qquad \dfrac{v_W^2}{60} = (0.22)(9.8) \qquad v_W = 11.37 \approx 11\ \text{m/s}$

c) $T = \dfrac{2\pi r}{v_W} = \dfrac{2\pi(60)}{11.37} = 33\ \text{s}$

6. $r_A = 64$ m, $a_{CA} = 8.0\ \text{m/s}^2$

a) $T_A = T_B = T$

$a_{CA} = \dfrac{4\pi^2 r_A}{T^2} \qquad 8.0 = \dfrac{4\pi^2(64)}{T^2} \qquad T = \sqrt{32}\,\pi \approx 17.8\ \text{s}$

b) $m_B = 52$ kg, $r_B = 48$ m

$a_{CB} = \dfrac{4\pi^2 r_B}{T^2} = \dfrac{4\pi^2(48)}{32\pi^2} = 6.0\ \text{m/s}^2$

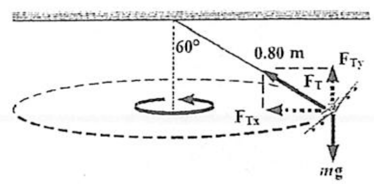
$F_{CB} = F_{NB} \qquad F_{NB} = m_B a_{CB} = (52)(6.0) = 312 \approx 310\ \text{N}$

c) $a_C = \dfrac{4\pi^2 r}{T^2} \qquad F_C = ma_C \qquad F_C = F_N$

If the period of the rotation decreases, the centripetal acceleration increases ($a_C \propto 1/T^2$) and so the centripetal force increases. Since in this case the normal force provides the centripetal force, the normal force experienced by the astronauts increases.

7. $L = 0.80$ m, $r = L \cdot \sin\theta = (0.80)(\sin 60°) = 0.69$ m

$F_{Tx} = F_T \sin 60°$ $\quad F_{Ty} = F_T \cos 60°$



a) $mg = F_{Ty}$ $\quad mg = F_T \cos 60°$ $\quad F_T = \dfrac{mg}{\cos 60°}$

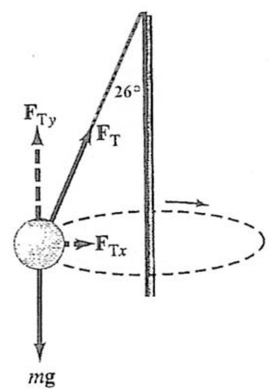
$F_C = F_{Tx}$ $\quad F_C = F_T \sin 60°$

$$\frac{mv^2}{r} = \left(\frac{mg}{\cos 60°}\right)\sin 60°$$

$$v = \sqrt{rg \cdot \tan 60°} = \sqrt{(0.69)(9.8)\tan 60°} = 3.4 \text{ m/s}$$

b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(0.69)}{3.4} = 1.3$ s

8. $F_{Tx} = F_T \sin 26°$ $\quad F_{Ty} = F_T \cos 26°$



a) $mg = F_{Ty}$ $\quad mg = F_T \cos 26°$ $\quad F_T = \dfrac{mg}{\cos 26°}$

$F_C = F_{Tx}$ $\quad F_C = F_T \sin 26°$

$$ma_C = \left(\frac{mg}{\cos 26°}\right)\sin 26°$$

$a_C = g \cdot \tan 26° = (9.8)\tan 26° = 4.8 \text{ m/s}^2$

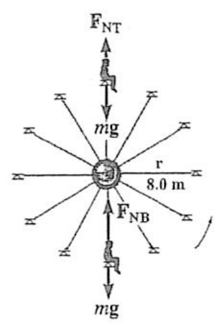
b) $T = 1.6$ s

$a_C = \dfrac{4\pi^2 r}{T^2}$ $\quad 4.8 = \dfrac{4\pi^2 r}{(1.6)^2}$ $\quad r = 0.31$ m

$a_C = \dfrac{v^2}{r}$ $\quad 4.8 = \dfrac{v^2}{0.31}$ $\quad v = 1.22$ m/s

9. $m = 40$ kg, $r = 8.0$ m, $T = 12$ s

a) Diagram showing the forces acting on the girl



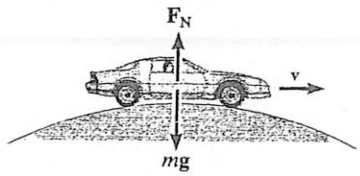
b) The direction of the center of the wheel is chosen as positive.

At the top: $F_{CT} = mg - F_{NT}$ $\quad F_{NT} = mg - F_{CT} = mg - m\dfrac{4\pi^2 r}{T^2} = (40)(9.8) - (40)\dfrac{4\pi^2(8.0)}{(12)^2} = 304 \approx 300$ N

At the bottom: $F_{CB} = F_{NB} - mg$ $\quad F_{NB} = F_{CB} + mg = m\dfrac{4\pi^2 r}{T^2} + mg = (40)\dfrac{4\pi^2(8.0)}{(12)^2} + (40)(9.8) = 480$ N

10. $m = 1200$ kg, $r = 48$ m

a) Diagram showing the forces acting on the vehicle



b) $v = 14$ m/s

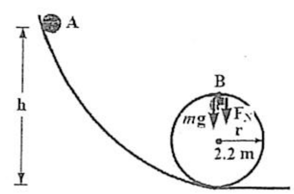
$F_C = mg - F_N \qquad F_N = mg - F_C = mg - \frac{mv^2}{r} = (1200)(9.8) - \frac{(1200)(14)^2}{48} = 6860 \approx 6900$ N

c) When $F_N = 0$, the vehicle can pass the top of the hill before losing contact with the road.

$F_C = mg - F_N \qquad \frac{mv^2}{r} = mg - 0 \qquad v = \sqrt{rg} = \sqrt{(48)(9.8)} = 21.69 \approx 22$ m/s

11. $m = 0.25$ kg

a) $r = 2.2$ m, $h_B = 2r = 2(2.2) = 4.4$ m



At point B, mg and F_N provide the centripetal force. The minimum speed for the sphere to remain on the track occurs when $F_N = 0$.

$F_C = mg + F_N \qquad \frac{mv_B{}^2}{r} = mg + 0 \qquad v_B{}^2 = gr$

Energy conservation: $KE_A + PE_A = KE_B + PE_B$

$0 + mgh_A = \frac{1}{2}mv_B{}^2 + mgh_B \qquad gh_A = \frac{1}{2}gr + g(2r) \qquad h_A = 2.5r = (2.5)(2.2) = 5.5$ m

b) $h_A = 11$ m, $r = 2.2$ m, $h_B = 4.4$ m

Energy conservation: $KE_A + PE_A = KE_B + PE_B$

$0 + mgh_A = \frac{1}{2}mv_B{}^2 + mgh_B \qquad (9.8)(11) = \frac{1}{2}v_B{}^2 + (9.8)(4.4)$

$v_B{}^2 = 129.36$ m^2/s^2 $\qquad v_B = 11.37 \approx 11$ m/s

$F_C = mg + F_N \qquad F_N = \frac{mv_B{}^2}{r} - mg = \frac{(0.25)(129.36)}{2.2} - (0.25)(9.8) = 12.25 \approx 12$ N

c) $h_A = 11$ m, $r = 1.1$ m, $h_B = 2.2$ m

Energy conservation: $KE_A + PE_A = KE_B + PE_B$

$0 + mgh_A = \frac{1}{2}mv_B{}^2 + mgh_B \qquad (9.8)(11) = \frac{1}{2}v_B{}^2 + (9.8)(2.2)$

$v_B{}^2 = 172.48$ m^2/s^2 $\qquad v_B = 13.13 \approx 13$ m/s

$F_C = mg + F_N \qquad F_N = \frac{mv_B{}^2}{r} - mg = \frac{(0.25)(172.48)}{1.1} - (0.25)(9.8) = 36.75 \approx 37$ N