Equilibrium

typical problems involve tension in wires supporting weights and torques produced by rigid objects such as beams

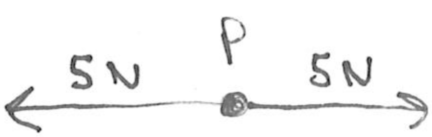
↙ ↘

Static (translational) Equilibrium

- ↙ not moving
- ↓ to move through space without rotating

Rotational Equilibrium

translational Equilibrium → the condition where the vector sum of all forces acting on a body is <u>zero</u>



The point P does not move - we say it is in static equilibrium → the sum of all forces acting on it is zero

VI. EQUILIBRIUM

POINT 1 Translational Equilibrium

A. Equilibrium

a. The complete motion of an object can be described as the translational motion of its center of mass and the rotational motion about its center of mass.

b. An object at rest or moving with constant velocity is in equilibrium. In equilibrium, its motion is unchanging and its acceleration is zero.

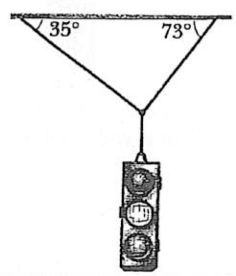
B. Translational equilibrium: The first condition for equilibrium

In translational equilibrium, the sum of the external forces is zero.

$\Sigma F = 0$: $\Sigma F_x = 0$, $\Sigma F_y = 0$

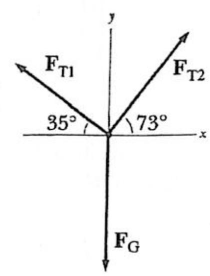
PROBLEM 1 Translational Equilibrium: $\Sigma F = 0$

A 15 kg traffic light is suspended by two cables as shown. Determine the tension in each cable.



Solution $m = 15$ kg, $F_G = mg = (15)(9.8) = 147$ N

Solution I: Component Method



Draw a free-body diagram.

Components:

$F_{T1x} = -F_{T1}\cos 35°$ $\quad F_{T1y} = F_{T1}\sin 35°$

$F_{T2x} = F_{T2}\cos 73°$ $\quad F_{T2y} = F_{T2}\sin 73°$

$F_{Gx} = 0$ $\quad F_{Gy} = mg = (15)(-9.8) = -147$ N

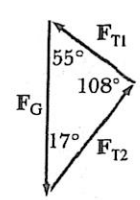
Translational equilibrium: $\Sigma F_x = 0$, $\Sigma F_y = 0$

$\Sigma F_x = F_{T1x} + F_{T2x} + F_{Gx} = -F_{T1}\cos 35° + F_{T2}\cos 73° + 0 = 0$

$\Sigma F_y = F_{T1y} + F_{T2y} + F_{Gy} = F_{T1}\sin 35° + F_{T2}\sin 73° - 147\text{ N} = 0$

$F_{T1} = 45.2 \approx 45$ N $\qquad F_{T2} = 126.6 \approx 130$ N

Solution II: Graphical Method



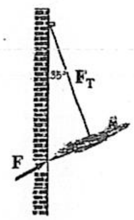
Draw a force triangle, and use a sine law.

$$\frac{F_G}{\sin 108°} = \frac{F_{T1}}{\sin 17°} \qquad F_{T1} = \frac{(147)\sin 17°}{\sin 108°} = 45.2 \approx 45\text{ N}$$

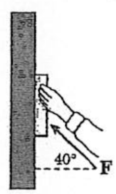
$$\frac{F_G}{\sin 108°} = \frac{F_{T2}}{\sin 55°} \qquad F_{T2} = \frac{(147)\sin 55°}{\sin 108°} = 126.6 \approx 130\text{ N}$$

RELATED PROBLEMS

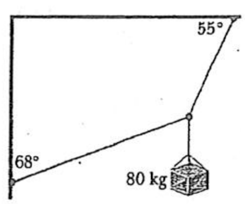
1. A 0.50 kg toy airplane is in equilibrium while it is hung using a string as shown in the figure. The tension force in the string is 3.9 N. Find the reaction force, F, exerted on the toy airplane by the wall.



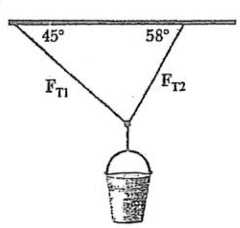
2. A student pushes with a minimum force of 50 N at an angle of 40° to a block to hold it in equilibrium. The coefficient of friction between the block and the wall is 0.28. What is the mass of the block?



3. An 80 kg crate is suspended from a ceiling and attached to a wall. Determine the tension in each rope.



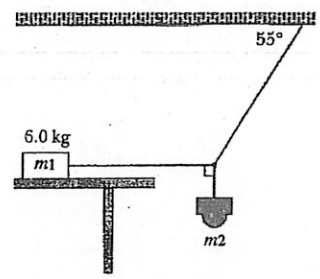
4. A bucket is suspended by two ropes as shown in the figure. The tension in the right rope (F_{T2}) is 180 N.



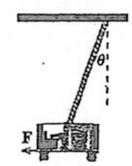
a) What is the tension in the left rope (F_{T1})?

b) What is the mass of the bucket?

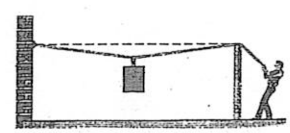
5. The system in the figure is in equilibrium. The coefficient of friction between the 6.0 kg block and the horizontal surface is 0.36. Find the maximum mass of m_2 that can be suspended by the ropes before the 6.0 kg block begins to slip:



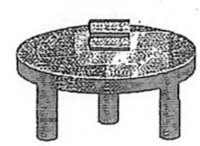
6. A student pulls horizontally on a toy car with force F so that the supporting rope makes an angle θ with the vertical. If the angle with the vertical increases, how will the tension force in the supporting rope change? Using principles of physics, explain your answer.



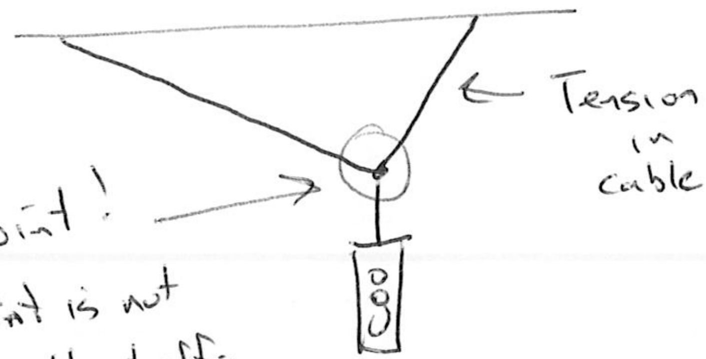
7. A crate hangs from a ring at the middle of a rope as shown in the figure. Is it possible for the man to make the rope perfectly horizontal? Using principles of physics, explain your answer.



8. Two books rest on a table as shown in the diagram. Using principles of physics, explain why the books are in translational equilibrium.



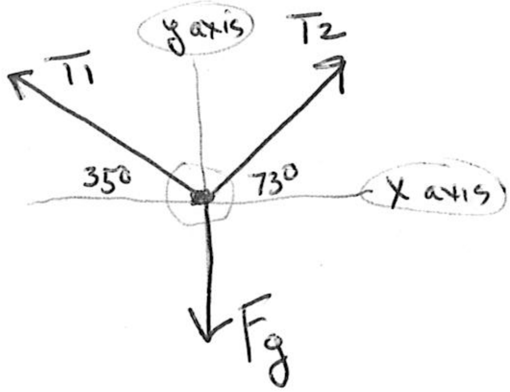
Problem 1 (see page 43)



Key point!

If this point is not moving then the traffic light is not moving

Step 1 → Draw a diagram showing the forces acting on the Key point!

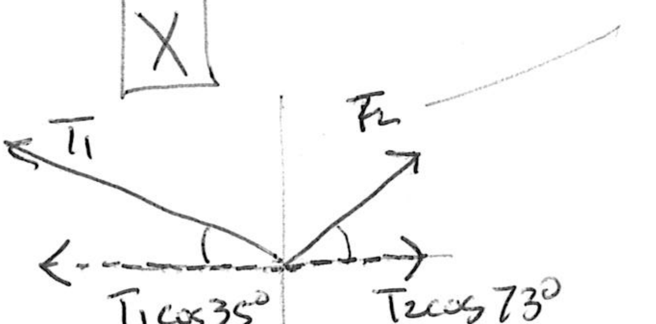


Step 2 Generate your equations using <u>components</u>

The sum of the x-components = 0

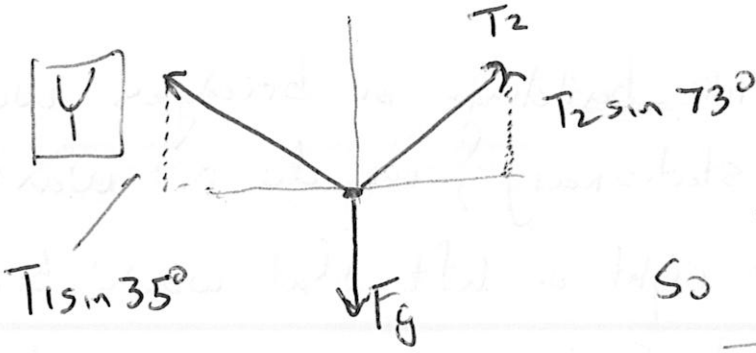
The sum of the y-components = 0

X



→ so $T_1 \cos 35^\circ = T_2 \cos 73^\circ$

Fg → has no x-component



So $T_1 \sin 35^\circ + T_2 \sin 73^\circ = F_g$

Each equation has two unknowns (T_1 and T_2) so neither equation can be solved! (on its own)

To solve — isolate a variable in one equation and substitute it into the other equation

$$T_1 = \frac{T_2(\cos 73^\circ)}{(\cos 35^\circ)}$$

$$T_1 = 0.357 T_2$$

$$T_1(\sin 35^\circ) + T_2(\sin 73^\circ) = mg$$

$$(0.357 T_2)(0.573) + 0.956 T_2 = 15(9.8)$$

$$0.205 T_2 + 0.956 T_2 = 147$$

$$1.16 T_2 = 147$$

$$T_2 = 127$$

now sub back into 1st equation

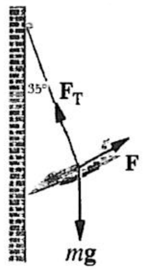
$$T_1 = \frac{127(\cos 73^\circ)}{(\cos 35^\circ)}$$

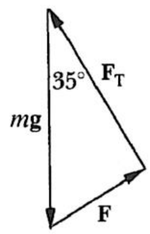
$$T_1 = 45\ N$$

$$T_1 = 45\ N$$
$$T_2 = 130\ N$$

VI. EQUILIBRIUM

1. $m = 0.50$ kg, $F_T = 3.9$ N

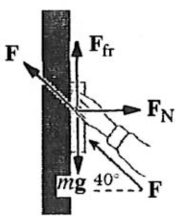


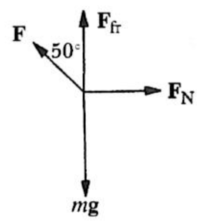


$mg = (0.50)(9.8) = 4.9$ N

$F = \sqrt{(mg)^2 + (F_T)^2 - 2(mg)(F_T)\cos 35^\circ} = \sqrt{(4.9)^2 + (3.9)^2 - 2(4.9)(3.9)\cos 35^\circ} = 2.8$ N

2. $F = 50$ N, $\mu = 0.28$

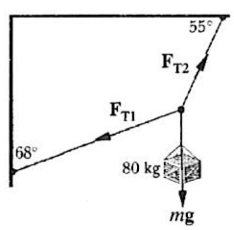


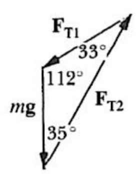


$F_N = F \sin 50^\circ = (50) \sin 50^\circ = 38.3$ N
$F_{fr} = \mu \cdot F_N = (0.28)(38.3) = 10.72$ N

$mg = F_{fr} + F \cos 50^\circ$ $(9.8)m = 10.72 + (50) \cos 50^\circ$ $m = 4.4$ kg

3. $m = 80$ kg

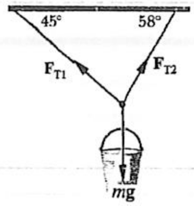


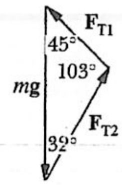


$\frac{mg}{\sin 33^\circ} = \frac{F_{T1}}{\sin 35^\circ}$ $F_{T1} = \frac{(80)(9.8)\sin 35^\circ}{\sin 33^\circ} = 825.7 \approx 830$ N

$\frac{mg}{\sin 33^\circ} = \frac{F_{T2}}{\sin 112^\circ}$ $F_{T2} = \frac{(80)(9.8)\sin 112^\circ}{\sin 33^\circ} = 1334.7 \approx 1300$ N

4. a) $F_{T2} = 180$ N

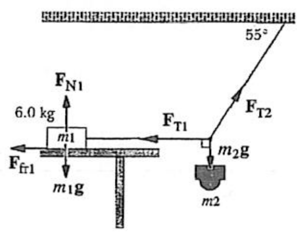


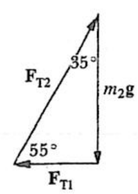


$$\frac{F_{T2}}{\sin 45°} = \frac{F_{T1}}{\sin 32°} \qquad F_{T1} = \frac{(180)\sin 32°}{\sin 45°} = 134.9 \approx 130 \text{ N}$$

b) $$\frac{F_{T2}}{\sin 45°} = \frac{mg}{\sin 103°} \qquad (9.8)m = \frac{(180)\sin 103°}{\sin 45°} \qquad m = 25 \text{ kg}$$

5. $m_1 = 6.0$ kg, $\mu = 0.36$

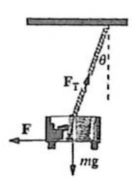


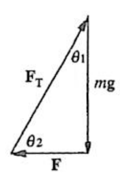


$F_{N1} = m_1g = (6.0)(9.8) = 58.8$ N
$F_{T1} = F_{fr1} = \mu \cdot F_{N1} = (0.36)(58.8) = 21.17$ N

$$\frac{m_2g}{\sin 55°} = \frac{F_{T1}}{\sin 35°} \qquad (9.8)m_2 = \frac{(21.17)\sin 55°}{\sin 35°} \qquad m_2 = 3.1 \text{ kg}$$

6. If θ_1 increases, θ_2 decreases and so the tension F_T increases because mg is constant.

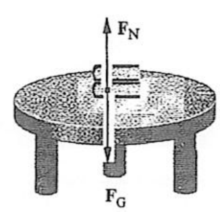




$$\frac{F_T}{\sin 90°} = \frac{mg}{\sin \theta_2} \qquad F_T = \frac{mg \sin 90°}{\sin \theta_2} = \frac{mg}{\sin \theta_2}$$

7. He cannot make the rope perfectly horizontal because the horizontal rope has no upward vertical component which produces an upward force to balance the weight of the crate.

8. The books are in translational equilibrium because the forces acting on it are balanced, and the net force on it is zero.



$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \quad (F_N - F_G = 0) \qquad \Sigma F = 0$