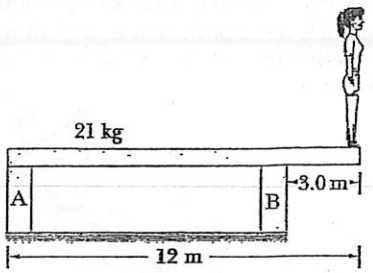
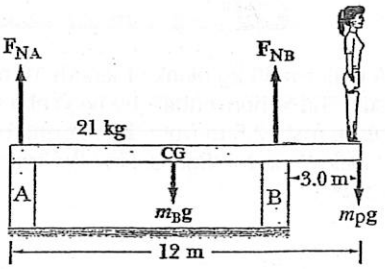
PROBLEM 4 Static Equilibrium: $\Sigma F = 0$, $\Sigma\tau = 0$

A uniform 12 m diving board with a mass of 21 kg rests on two supports as shown in the diagram. A diver is standing at the right end of the board. If support B exerts a reaction force of 960 N on the board, what reaction force does support A exert?



Solution $m_B = 21$ kg, $F_{NB} = 960$ N



Draw a free-body diagram.

Solution I: Using two pivot points
In rotational equilibrium, the sum of the torques about any pivot point is zero.

pivot: support A
$d_L = 12$ m, $d_B = 9.0$ m, $d_{CG} = 6.0$ m

Rotational equilibrium: $\Sigma\tau = 0$
$\Sigma\tau = \tau_{CC} - \tau_C = (F_{NB})(d_B) - [(m_Bg)(d_{CG}) + (m_Pg)(d_L)] = 0$
$(960)(9.0) - [(21)(9.8)(6.0) + (m_P)(9.8)(12)] = 0$ $\quad m_P = 63$ kg

Pivot: support B
$d_{CG} = 3.0$ m, $d_A = 9.0$ m, $d_P = 3.0$ m

Rotational equilibrium: $\Sigma\tau = 0$
$\Sigma\tau = \tau_{CC} - \tau_C = (m_Bg)(d_{CG}) - [(F_{NA})(d_A) + (m_Pg)(d_P)] = 0$
$(21)(9.8)(3.0) - [(F_{NA})(9.0) + (63)(9.8)(3.0)] = 0$ $\quad F_{NA} = -137.2 \approx -140$ N

Solution II: Using the conditions for static equilibrium: $\Sigma\tau = 0$, $\Sigma F = 0$

pivot: support A
$d_L = 12$ m, $d_B = 9.0$ m, $d_{CG} = 6.0$ m

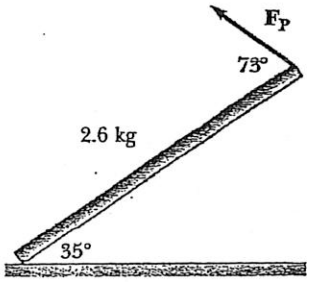
Rotational equilibrium: $\Sigma\tau = 0$
$\Sigma\tau = \tau_{CC} - \tau_C = (F_{NB})(d_B) - [(m_Bg)(d_{CG}) + (m_Pg)(d_L)] = 0$
$(960)(9.0) - [(21)(9.8)(6.0) + (m_P)(9.8)(12)] = 0$ $\quad m_P = 63$ kg

Translational equilibrium: $\Sigma F = 0$
$\Sigma F_y = [F_{NA} + F_{NB}] - [m_Bg + m_Pg] = 0$
$(F_{NA} + 960) - [(21)(9.8) + (63)(9.8)] = 0$ $\quad F_{NA} = -136.8 \approx -140$ N

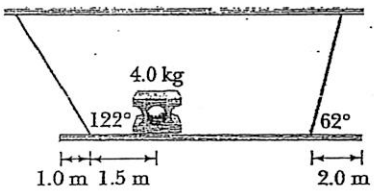
Note: The negative sign of F_{NA} means that the reaction force of support A actually points downward. In other words, support A indeed pulls downward on the diving board by means of bolts or screws.

RELATED PROBLEMS

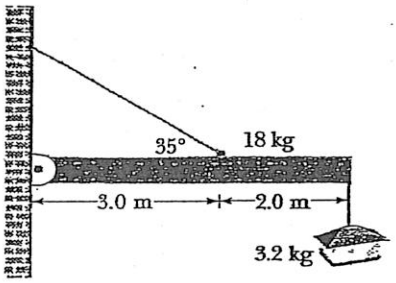
19. A uniform 2.6 kg beam held by the force F_P is about to slide to the left as shown in the figure. Find the force of friction acting on the beam by the ground.



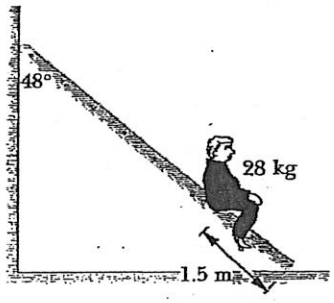
20. A uniform 50 kg plank of length 10 m is suspended horizontally by two cables. A 4.0 kg block rests 2.5 m from the left end of the plank as shown in the figure. Find the tension in each cable.



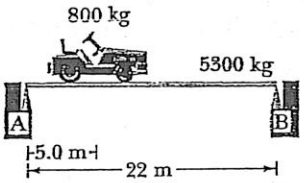
21. A uniform 18 kg beam with a length of 5.0 m is mounted by a hinge on a wall. The beam is held in a horizontal position by a cable with an angle of 35° and supports a cage with a mass of 3.2 kg as shown in the figure. Find the force that the hinge exerts on the beam.



22. A uniform ladder of length 5.0 m and mass 14 kg rests against a smooth, vertical wall. The term "smooth" means that the wall can exert only a normal force directed perpendicular to the wall and cannot exert a frictional force parallel to it. A 28 kg boy is sitting on a step 1.5 m from the bottom of the ladder. Find the coefficient of friction between the ladder and the ground.



23. An 800 kg automobile moves toward the right end of the bridge which has a mass of 5300 kg and a length of 22 m.



a) Find the reaction forces exerted by supports A and B on the bridge at the position shown in the diagram.

b) As the automobile approaches support B, what will the reaction forces exerted by support A and B experience?